Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #5

Anti-derivatives

When $f'(t) = g(t)$
we say that $f(t)$ is the **anti-derivative** of $g(t)$.

Examples:

$\frac{1}{3}x^3$ is the anti-derivative of x^2 because $\left(\frac{1}{3}x^3\right)' = x^2$

The function $\tan^{-1} x$ is the anti-derivative of $\frac{1}{x^2+1}$ because $(\tan^{-1} x)' = \frac{1}{x^2+1}$

Three Points:

I.

If $f' = g$ then also $(f + c)' = g$ for any constant c.

For example, $\left(\frac{1}{3}x^3 + 11\right)' = x^2$.

Whenever you find an anti-derivative, you can add any constant to it and still have an anti-derivative of the same function

II.

The definite integral

$$\int_{x=a}^{x=b} x^2 \, dx = \left(\frac{1}{3}b^3\right) - \left(\frac{1}{3}a^3\right)$$

doesn't change if you add a constant to the anti-derivative:

$$\int_{x=a}^{x=b} x^2 \, dx = \left(\frac{1}{3}b^3 + c\right) - \left(\frac{1}{3}a^3 + c\right)$$

The constant always cancels out.

III. Notation

We write

$$\int f(x)\,dx$$

to stand for any anti-derivative of the function $f(x)$.

So:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$$

Because

$$(\tan^{-1} x + c)' = \frac{1}{1+x^2}$$

for any constant c.

Examples:

1. Suppose $\int f(x)\,dx = \sin x + c$ then

$$\int_0^{\pi} f(x)\,dx = \sin \pi - \sin 0$$

2. Suppose $\int \log x\,dx = g(x) + \ c$ then

$$\int_1^{e} \log x\,dx = g(e) - \ g(1)$$

3. A favourite trick question: Evaluate

$$\int_{-1}^{+1} (e^{x^2})'\ dx$$

The point is that e^{x^2} is an anti-derivative of its own derivative! So the question gives us the whole answer:

$$\int_{-1}^{+1} (e^{x^2})'\ dx = e^{1^2} - e^{(-1)^2} = 0$$